Momentum & Impulse

Hard Practice — Standard 5.1 (Double Standard)

Impulse–momentum theorem • conservation • elastic & perfectly inelastic • 1D and 2D collisions • explosions • force / momentum / velocity vs time graphs

Sign convention: right / east = positive, left = negative. Use g = 9.8 m/s^2. Show all work. Diagrams are provided where a problem references one.

Part A — Momentum, Impulse, and the Force–Time Trade-Off

Problem 1.

A 1420 kg car travels at 25 m/s due east. On the same road, a 68 kg cyclist travels at 12 m/s due west. Treating both as one system, find the total momentum of the system: magnitude and direction.

Problem 2.

A 0.145 kg baseball approaches a bat at 38 m/s. It leaves the bat at 46 m/s in the exact opposite direction. Bat contact lasts 0.70 ms. Find (a) the impulse delivered to the ball, and (b) the average force the bat exerts.

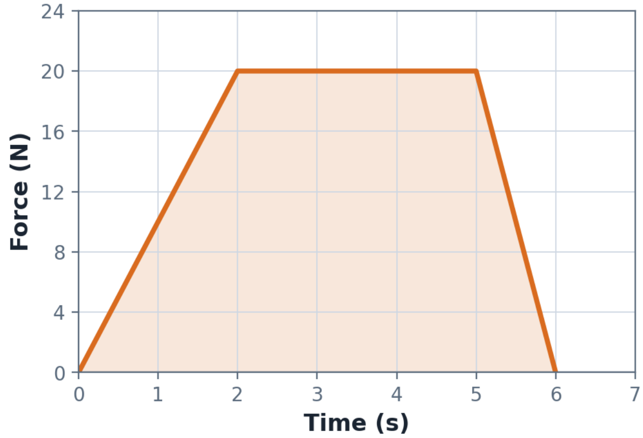
Problem 3.

A 1200 kg car moving at 28 m/s is brought to a complete stop in a crash. Find the average force on the car if the stop takes (a) 0.080 s against a rigid wall, and (b) 1.2 s in a controlled crumple. Then state, using $J = F\Delta t$, why the crumple case is survivable.

Part B — Force vs Time Graphs

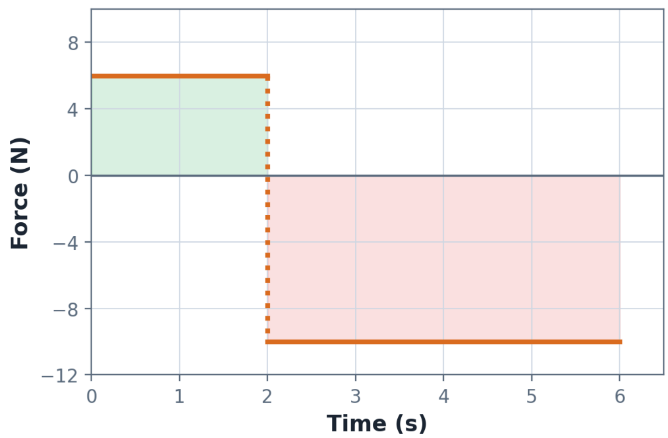
Problem 4.

The force vs time graph below acts on a 4.0 kg object initially at rest. Find (a) the total impulse delivered, and (b) the object's final velocity.



Problem 5.

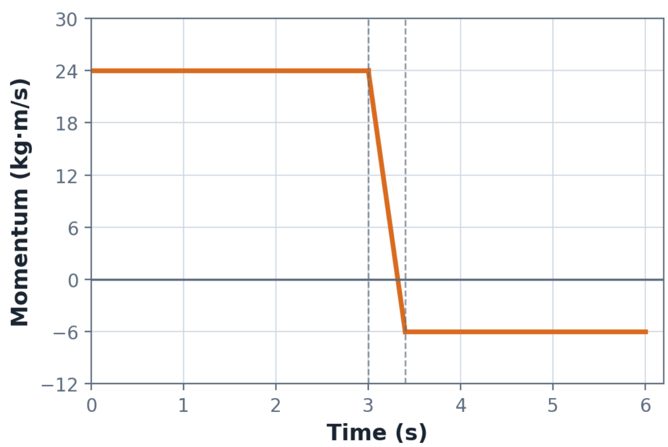
The force vs time graph below acts on a 2.0 kg object that starts moving at +10 m/s. Find (a) the net impulse over the full interval, (b) the object's final velocity, and (c) the exact time at which the object is momentarily at rest.



Part C — Momentum & Velocity vs Time Graphs

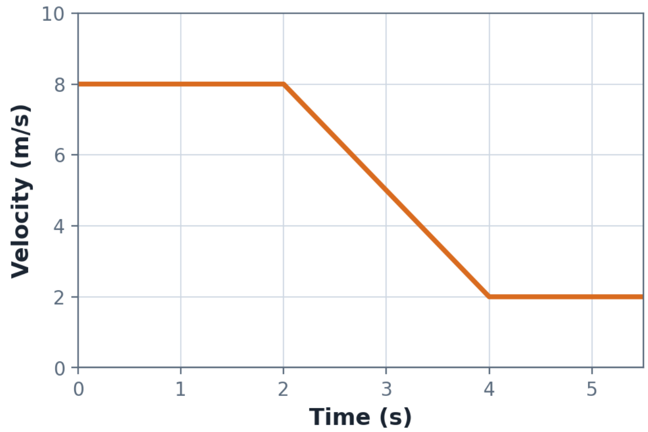
Problem 6.

The momentum vs time graph below describes a single object during a collision. Determine (a) the instant the collision begins, (b) its duration, (c) the change in momentum, (d) the average force during the collision, and (e) qualitatively, what happens to the object's direction of motion.



Problem 7.

The velocity vs time graph below is for a 3.0 kg object. Find (a) its momentum at t = 0 and at t = 4 s, (b) the impulse delivered between t = 2 s and t = 4 s, and (c) the average force over that interval.



Part D — Perfectly Inelastic Collisions (1D)

Problem 8.

A 1500 kg car moving at 20 m/s rear-ends a stationary 1000 kg car. They lock together and move off as one. Find (a) their common velocity, and (b) the fraction of the original kinetic energy lost in the collision.

Problem 9.

A 12 g bullet is fired horizontally into a 2.0 kg wooden block hanging at rest from a string (a ballistic pendulum). The bullet embeds, and the block + bullet swing up to a maximum height of 0.18 m above the starting position. Find the bullet's initial speed.

Problem 10.

A 5.0 kg fish swims to the right at 1.0 m/s and swallows a 2.0 kg fish swimming to the left at 2.0 m/s. Find the velocity (magnitude and direction) of the combined fish. A classmate's chart claims the total momentum is 9 kg·m/s. Is that right? Explain the error in one sentence.

Part E — Elastic Collisions (1D)

Problem 11.

A 3.0 kg cart moving at 4.0 m/s collides elastically with a 1.0 kg cart at rest. Find both final velocities, then verify your answer by checking that kinetic energy is conserved.

Problem 12.

A 2.0 kg glider moving right at 5.0 m/s collides head-on and elastically with a 4.0 kg glider moving left at 3.0 m/s. Find both final velocities, with direction.

Part F — Explosions & Recoil

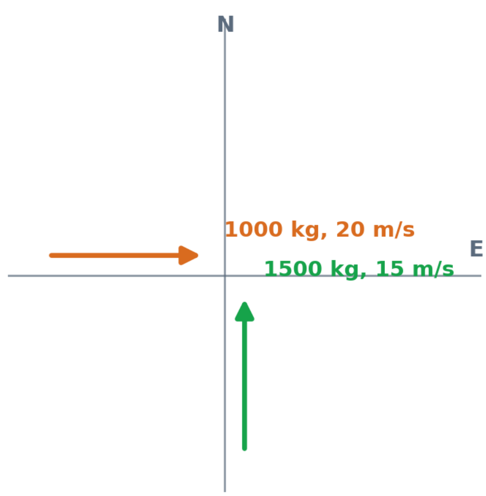
Problem 13.

A 3.0 kg cart sits at rest on frictionless track. An internal spring releases, splitting it into a 1.0 kg piece that moves left at 6.0 m/s and a 2.0 kg piece. Find the velocity of the 2.0 kg piece.

Part G — Two-Dimensional Collisions (Level-4 Ceiling)

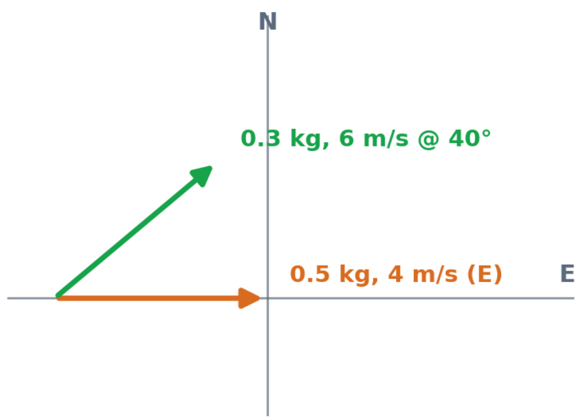
Problem 14.

A 1000 kg car moving east at 20 m/s collides with a 1500 kg car moving north at 15 m/s. They stick together. Find the speed and direction of the wreckage.



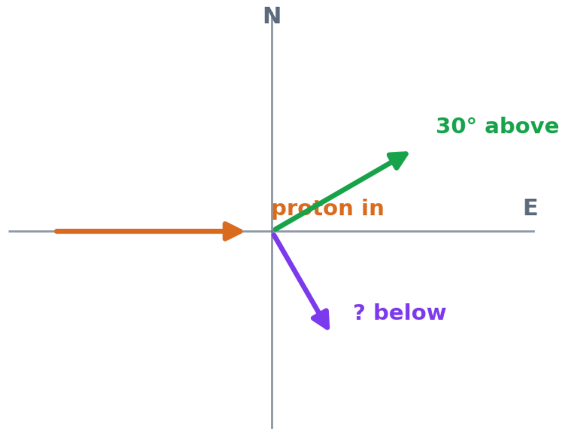
Problem 15.

A 0.50 kg puck moving east at 4.0 m/s strikes and sticks to a 0.30 kg puck moving at 6.0 m/s in a direction 40° north of east. Find the speed and direction of the combined pucks.



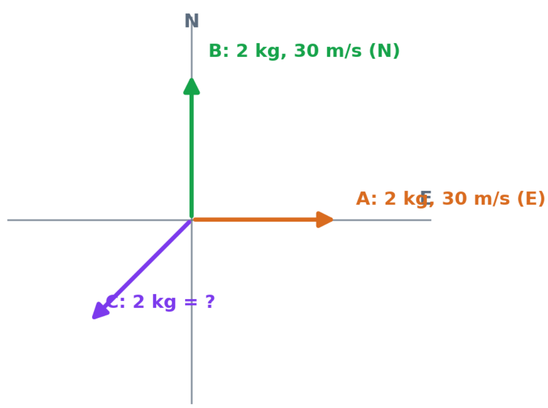
Problem 16.

A proton moving at 4.0×10^5 m/s strikes a second, stationary proton in an elastic collision. Afterward, the first proton moves off at 30° above its original path. Using the fact that two equal masses in an elastic 2D collision separate at 90°, find the direction and speed of BOTH protons after the collision.



Problem 17.

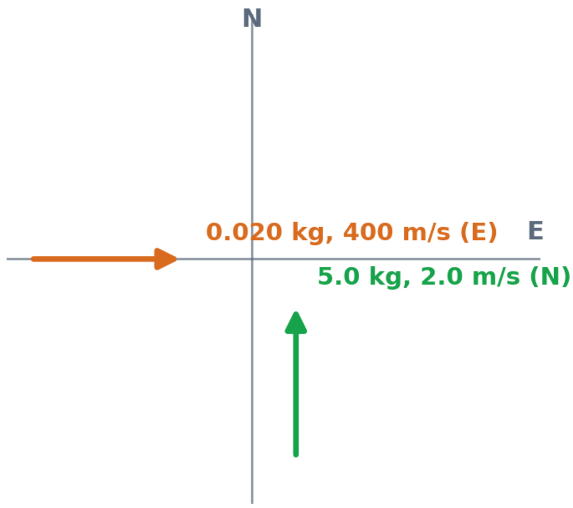
A 6.0 kg object at rest explodes into three 2.0 kg fragments that all move in one horizontal plane. Fragment A moves east at 30 m/s; fragment B moves north at 30 m/s. Find the velocity (magnitude and direction) of fragment C.



Part H — Multi-Stage Synthesis

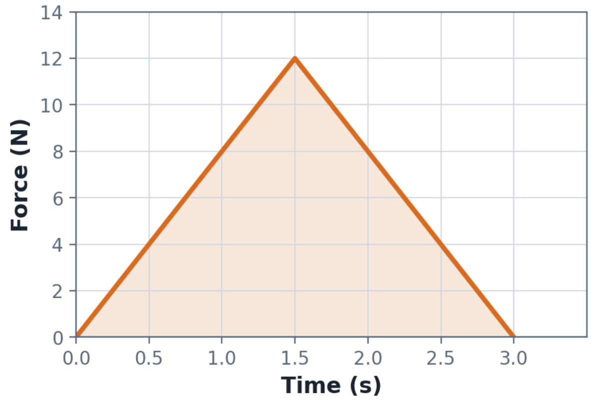
Problem 18.

A 0.020 kg bullet moving east at 400 m/s embeds in a 3.0 kg block resting on frictionless ice. The block + bullet then slide east and collide with a 5.0 kg block moving north at 2.0 m/s; these two stick together. Find the final velocity (speed and direction) of the entire mass.



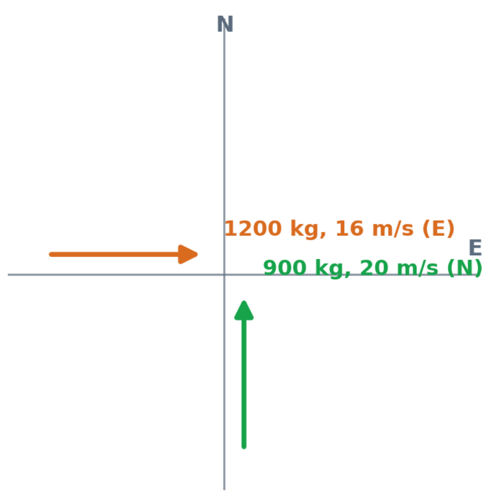
Problem 19.

A 2.0 kg cart starts at rest. The force vs time graph below acts on it, after which it collides elastically with a 4.0 kg cart at rest. Find (a) the speed the first cart reaches from the graph, and (b) both cart velocities after the elastic collision.



Problem 20.

A 1200 kg car moving east at 16 m/s collides with a 900 kg car moving north at 20 m/s. They stick together. Find (a) the wreckage's speed and direction, and (b) the percentage of the system's kinetic energy lost in the collision.



Answer Key — Full Worked Solutions

Problem 1

Car: $p = mv = (1420)(25) = +35\,500$ kg·m/s (east).

Cyclist: west is negative, $p = (68)(12) = 816$, so -816 kg·m/s.

Total: $35\,500 + (-816) = 34\,684$ kg·m/s.

Answer: 34 684 kg·m/s, directed east.

Common error: Adding the magnitudes (giving 36 316). West must enter as a negative momentum.

Problem 2

Let the leaving direction be +. Then $v_i = -38$ m/s, $v_f = +46$ m/s.

$\Delta p = m(v_f - v_i) = 0.145(46 - (-38)) = 0.145(84) = 12.18$ kg·m/s.

Impulse $J = \Delta p = 12.18$ N·s.

$F = J / \Delta t = 12.18 / 0.00070 = 17\,400$ N.

Answer: $J = 12.18$ N·s; $F \approx 1.74 \times 10^4$ N.

Common error: Using $46 - 38 = 8$ instead of $46 - (-38) = 84$. The reversal is the whole point.

Problem 3

$\Delta p = m(v_f - v_i) = 1200(0 - 28) = -33\,600$ kg·m/s. This is the same for both cases.

(a) Wall: $F = 33\,600 / 0.080 = 420\,000$ N.

(b) Crumple: $F = 33\,600 / 1.2 = 28\,000$ N.

Same Δp, 15× longer stop → 15× smaller force on the occupants.

Answer: Wall $F = 4.2 \times 10^5$ N; crumple $F = 2.8 \times 10^4$ N.

Common error: Recomputing Δp differently for the two parts. Δp is fixed by the situation; only Δt and F trade.

Problem 4

Impulse = area under the graph.

Ramp up (0→2 s): ½(2)(20) = 20. Hold (2→5 s): (3)(20) = 60. Ramp down (5→6 s): ½(1)(20) = 10.

Total impulse = 20 + 60 + 10 = 90 N·s.

v_f = Δp / m = 90 / 4.0 = 22.5 m/s.

> **Answer: Impulse = 90 N·s; v_f = 22.5 m/s.**

> **Common error:** Counting only the rectangle (60 N·s) and dropping the two triangular ends.

Problem 5

Impulse = signed area. First block: (+6)(2) = +12 N·s. Second block: (−10)(4) = −40 N·s.

Net impulse = 12 − 40 = −28 N·s.

p_i = (2)(10) = 20. p_f = 20 + (−28) = −8 kg·m/s → v_f = −4.0 m/s.

At rest when p = 0. At t = 2 s, p = 20 + 12 = 32. Then −10 N drains it: 0 = 32 − 10(t − 2) → t = 5.2 s.

> **Answer: Net impulse = −28 N·s; v_f = −4.0 m/s (reversed); at rest at t = 5.2 s.**

> **Common error:** Ignoring the +10 m/s starting momentum and reporting −28/2 = −14 m/s as the final velocity.

Problem 6

The graph is flat until it suddenly steepens — that steep segment is the collision.

(a) Collision begins at t = 3.0 s. (b) Duration = 3.4 − 3.0 = 0.4 s.

(c) Δp = p_f − p_i = (−6) − (24) = −30 kg·m/s.

(d) F = Δp / Δt = −30 / 0.4 = −75 N.

(e) Momentum goes from positive to negative → the object reverses direction.

> **Answer: Collision at t = 3.0 s, lasting 0.4 s; Δp = −30 kg·m/s; F = 75 N opposing motion; object reverses.**

> **Common error:** Reading Δp as the final value (−6) instead of the change (−30).

Problem 7

Momentum p = mv. (a) p(0) = 3(8) = 24 kg·m/s; p(4) = 3(2) = 6 kg·m/s.

(b) Impulse (2→4 s) = Δp = 6 − 24 = −18 kg·m/s (velocity is still 8 at t = 2 s).

(c) F = Δp / Δt = −18 / (4 − 2) = −9.0 N.

> **Answer: p(0) = 24 kg·m/s, p(4) = 6 kg·m/s; impulse = −18 N·s; F = 9.0 N opposing motion.**

> **Common error:** Reading impulse straight off the velocity axis without multiplying by the 3 kg mass.

Problem 8

Perfectly inelastic: $m_1v_1 + m_2v_2 = (m_1+m_2)v_f$.

$(1500)(20) + 0 = (2500)v_f \rightarrow v_f = 30\,000 / 2500 = 12$ m/s.

$KE_i = ½(1500)(20)^2 = 300\,000$ J. $KE_f = ½(2500)(12)^2 = 180\,000$ J.

Lost = 120 000 J → fraction = 120 000 / 300 000 = 0.40.

Answer: v_f = 12 m/s; 40% of the kinetic energy is lost.

Common error: Assuming kinetic energy is conserved. In a perfectly inelastic collision it is not — only momentum is.

Problem 9

Stage 1 (collision, momentum only): (0.012)v = (2.012)V.

Stage 2 (swing up, energy): $½(2.012)V^2 = (2.012)g\,h \rightarrow V = \sqrt{2gh} = \sqrt{2·9.8·0.18} = \sqrt{3.528} = 1.878$ m/s.

Back-substitute: v = (2.012)(1.878) / 0.012 = 3.779 / 0.012 = 314.9 m/s.

Answer: v ≈ 315 m/s.

Common error: Using energy conservation for the collision stage. The embedding is inelastic; KE is not conserved there — only in the swing.

Problem 10

Right is +. Big fish: p = (5.0)(+1.0) = +5. Small fish: moving left → p = (2.0)(−2.0) = −4.

Total p = 5 + (−4) = +1 kg·m/s. Combined mass = 7.0 kg.

v_f = 1 / 7 = 0.14 m/s to the right.

Answer: 0.14 m/s to the right (total p = 1 kg·m/s, not 9).

Common error: The classmate added |5| + |4| = 9, ignoring direction. The small fish's momentum is −4, not +4.

Problem 11

Elastic, target at rest: $v_1' = ((m_1-m_2)/(m_1+m_2))v_1$, $v_2' = (2m_1/(m_1+m_2))v_1$.

$v_1' = ((3-1)/4)(4) = 2.0$ m/s. $v_2' = (6/4)(4) = 6.0$ m/s.

Check p: 3(4)=12; after 3(2)+1(6)=12. ✓

Check KE: ½(3)(16)=24; after ½(3)(4)+½(1)(36)=6+18=24. ✓

Answer: v_1' = 2.0 m/s, v_2' = 6.0 m/s (both forward).

Common error: Assuming equal masses swap velocities — that shortcut only holds when the masses are equal.

Problem 12

Use both elastic formulas. m_1=2, v_1=+5; m_2=4, v_2=−3; sum = 6.

v_1' = ((2−4)/6)(5) + (2·4/6)(−3) = −10/6 − 24/6 = −34/6 = −5.67 m/s.

v_2' = (2·2/6)(5) + ((4−2)/6)(−3) = 20/6 − 6/6 = 14/6 = +2.33 m/s.

Check p: before 2(5)+4(−3) = −2; after 2(−5.67)+4(2.33) = −2. ✓

Answer: v_1' ≈ −5.67 m/s (bounces left), v_2' ≈ +2.33 m/s (right).

Common error: Dropping the sign on v_2 = −3 m/s in the second term of each formula.

Problem 13

System starts at rest → total momentum stays 0.

0 = (1.0)(−6.0) + (2.0)v → 2v = 6 → v = +3.0 m/s.

Answer: 3.0 m/s to the right.

Common error: Setting the total equal to some incoming momentum instead of 0. An explosion from rest keeps total p = 0.

Problem 14

Conserve each axis. p_x = (1000)(20) = 20 000 (E). p_y = (1500)(15) = 22 500 (N).

Combined mass = 2500. v_x = 8.0 m/s, v_y = 9.0 m/s.

Speed = $\sqrt{(8^2+9^2)}$ = √145 = 12.04 m/s. Angle = $\tan^{-1}(9/8)$ = 48.4° N of E.

Answer: 12.0 m/s at 48.4° north of east.

Common error: Adding the two speeds or momenta as scalars. Perpendicular momenta must combine by the Pythagorean theorem.

Problem 15

Puck 1: p_x = (0.5)(4) = 2.0, p_y = 0.

Puck 2: p_x = (0.3)(6)cos40° = 1.8(0.766) = 1.379; p_y = 1.8(0.643) = 1.157.

Total: p_x = 3.379, p_y = 1.157. Mass = 0.80 kg.

v_x = 4.224, v_y = 1.446. Speed = √(17.84+2.09) = 4.46 m/s. Angle = $\tan^{-1}$(1.446/4.224) = 18.9°.

Answer: 4.46 m/s at 18.9° north of east.

Common error: Failing to resolve puck 2 into components before adding. A 40° velocity contributes to both axes.

Problem 16

Equal masses, elastic, one at rest → the two leave 90° apart, so proton 2 goes 60° below the line.

For a deflection angle θ: $v_1' = v\cos\theta$ and $v_2' = v\sin\theta$.

$v_1' = (4.0\times10^5)\cos30° = 3.46\times10^5$ m/s at 30° above.

$v_2' = (4.0\times10^5)\sin30° = 2.0\times10^5$ m/s at 60° below.

Answer: Proton 1: 3.46×10^5 m/s at 30° above; Proton 2: 2.0×10^5 m/s at 60° below.

Common error: Forgetting the 90° separation result and trying to guess the second angle, or splitting the incoming speed evenly.

Problem 17

Total momentum = 0. p_A = (2)(30) = 60 (E). p_B = (2)(30) = 60 (N).

p_C must cancel both: p_C = (−60, −60) kg·m/s.

$v_C = p_C/2$ = (−30, −30) m/s. Speed = $\sqrt{(30^2+30^2)} = \sqrt{1800}$ = 42.4 m/s.

Direction = 45° south of west (down and to the left).

Answer: 42.4 m/s directed 45° south of west.

Common error: Treating momentum as a scalar and adding 30 + 30 = 60. The fragments' momenta are vectors that must sum to zero.

Problem 18

Stage 1 (bullet + block, 1D east): (0.020)(400) = $(3.02)v_1$ → v_1 = 8.0/3.02 = 2.65 m/s (E).

Stage 2 (2D, sticks to north-moving block): p_x = (3.02)(2.65) = 8.0 (E); p_y = (5.0)(2.0) = 10 (N).

Total mass = 8.02 kg. v_x = 8.0/8.02 = 0.998; v_y = 10/8.02 = 1.247.

Speed = $\sqrt{(0.995+1.555)} = \sqrt{2.55}$ = 1.60 m/s. Angle = $\tan^{-1}(1.247/0.998)$ = 51.3° N of E.

Answer: 1.60 m/s at 51.3° north of east.

Common error: Losing the bullet's momentum in stage 1, or merging the x and y momenta in stage 2 instead of keeping them separate.

Problem 19

(a) Impulse from the triangle = ½(3.0)(12) = 18 N·s → v = 18/2.0 = 9.0 m/s.

(b) Elastic, 2 kg at 9 into 4 kg at rest: v_1' = ((2−4)/6)(9) = −3.0 m/s; v_2' = (2·2/6)(9) = 6.0 m/s.

Check p: 2(9)=18; after 2(−3)+4(6)=18. ✓ KE: ½(2)(81)=81; after ½(2)(9)+½(4)(36)=81. ✓

Answer: Cart reaches 9.0 m/s; after collision v_1' = −3.0 m/s (rebounds), v_2' = 6.0 m/s.

Common error: Computing the triangle area as base×height instead of ½ base×height, or treating the collision as inelastic.

Problem 20

p_x = (1200)(16) = 19 200 (E). p_y = (900)(20) = 18 000 (N). Mass = 2100 kg.

v_x = 9.14, v_y = 8.57. Speed = $\sqrt{(83.6+73.5)} = \sqrt{157.1}$ = 12.5 m/s. Angle = $\tan^{-1}(8.57/9.14)$ = 43.2° N of E.

$KE_i = \frac{1}{2}(1200)(16^2) + \frac{1}{2}(900)(20^2)$ = 153 600 + 180 000 = 333 600 J.

$KE_f = \frac{1}{2}(2100)(157.1)$ = 164 900 J. Lost = 168 700 J → 168 700/333 600 = 0.506.

Answer: 12.5 m/s at 43.2° north of east; about 50.6% of the kinetic energy is lost.

Common error: Computing KE_f from v_x and v_y separately and adding, instead of using the single resultant speed ($v^2 = v_x^2 + v_y^2$).