Standard 4.2

Conservation of Energy — Hard Practice Set

Standard 4.2 uses multiple forms of energy and work to solve for unknowns in a closed system. Its rubric is a ladder: energy bar charts at the approaching tier, a familiar multistep problem at meeting, and a multistep problem you have never seen before at exceeding. This set climbs the whole ladder. Bar charts come first to make the energy visible, then the multistep chains combine gravity, springs, friction, and connected masses, ending on problems built to be unfamiliar. The one equation behind all of it:

MEDIUM HARD DIABOLICAL

$$W_{friction} + KE_i + PE_i + PE_{spring,i} = KE_f + PE_f + PE_{spring,f}$$

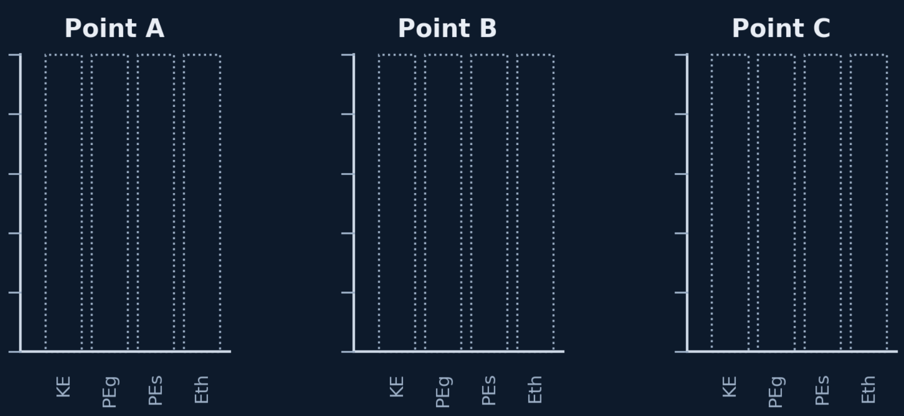
Whatever energy seems to “disappear” to friction has become thermal energy. Use g = 9.8 m/s^2.

MEDIUM HARD DIABOLICAL

Energy Bar Charts — See the Energy Move

MEDIUM

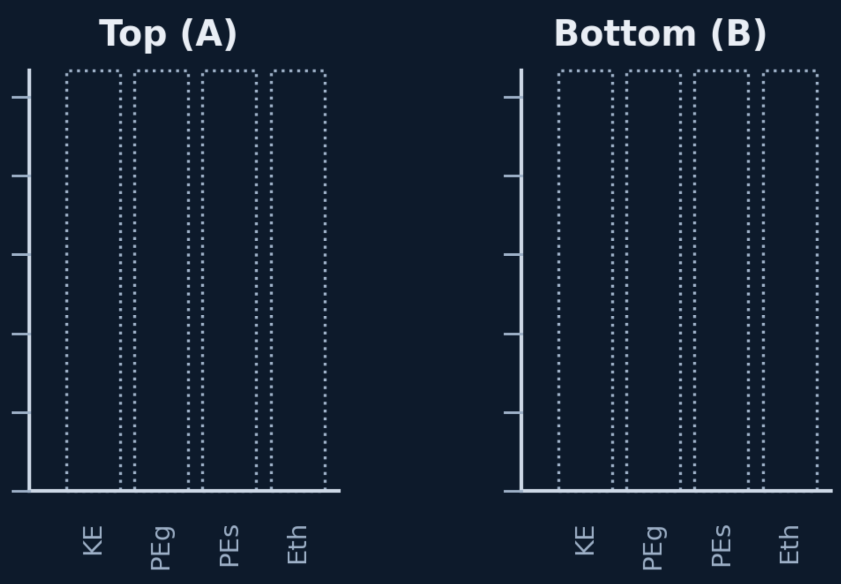
1. A 0.5 kg ball rolls on a frictionless track. It is released from rest at Point A (height 1.8 m), passes the lowest point B, and rises to Point C (height 1.0 m). (a) On the blank charts below, draw the bars for kinetic energy (KE), gravitational PE (PEg), spring PE (PEs), and thermal energy (Eth) at A, B, and C. (b) Find the ball's speed at B.



Fill each bar. Frictionless.

MEDIUM

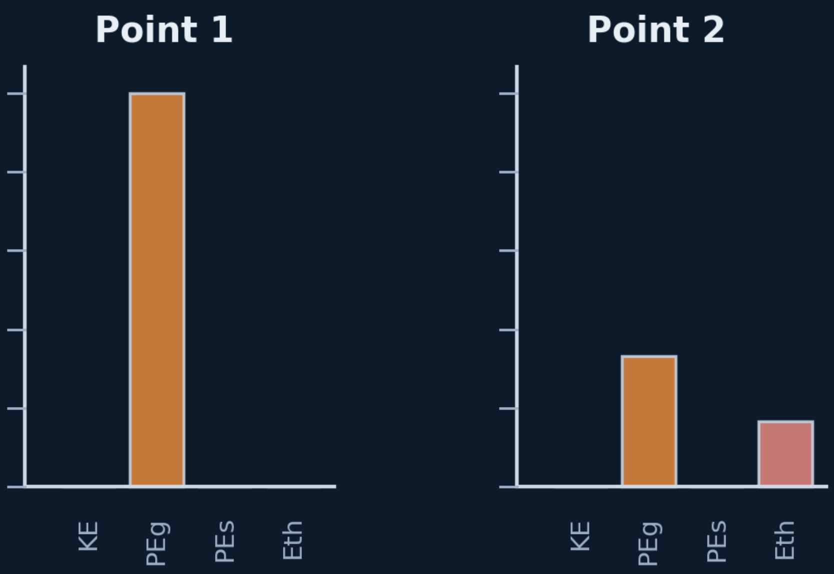
2. A 2.0 kg block slides down a rough ramp. It starts from rest at the top (A, height 1.5 m) and reaches the bottom (B) moving at 4.0 m/s. (a) Draw the bar charts at A and B, including thermal energy. (b) How much thermal energy was generated by friction?



Fill each bar. Rough ramp.

HARD

3. The two bar charts below show the energy of a 3.0 kg cart at two points on its path. All the energy at Point 1 is gravitational (60 J). At Point 2, gravitational PE is 20 J and 10 J has turned to thermal energy. (a) Find the kinetic energy at Point 2. (b) Find the cart's speed at Point 2.

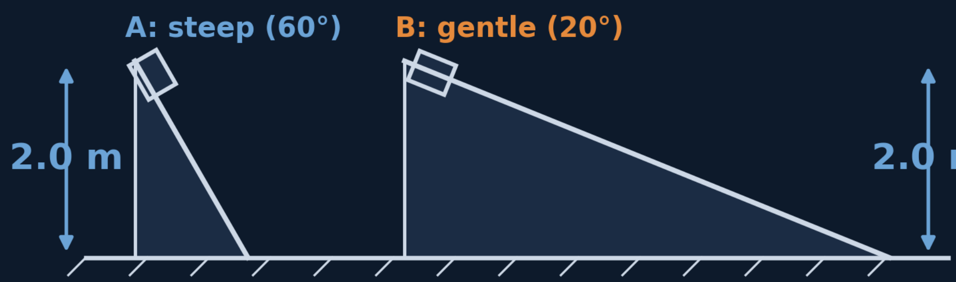


KE at Point 2 is unknown.

Same Height, Same Speed — but Not Same Time

HARD

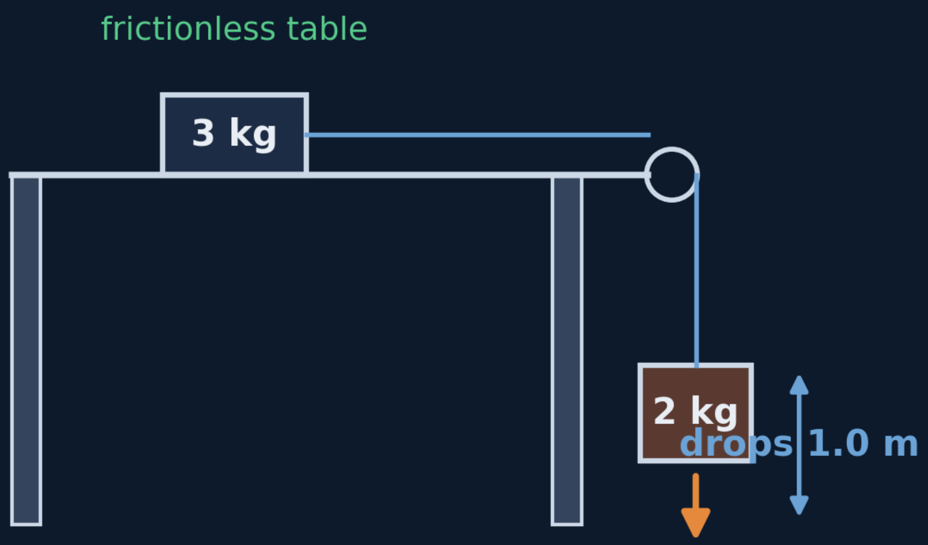
4. Two identical 1.0 kg blocks are released from rest at the top of two frictionless ramps of the same height (2.0 m): Ramp A is steep (60°), Ramp B is gentle (20°). (a) Find each block's speed at the bottom. (b) Are the speeds equal? (c) Do the blocks reach the bottom at the same time? Explain using energy vs. acceleration.



both 1.0 kg, released from rest, frictionless

MEDIUM

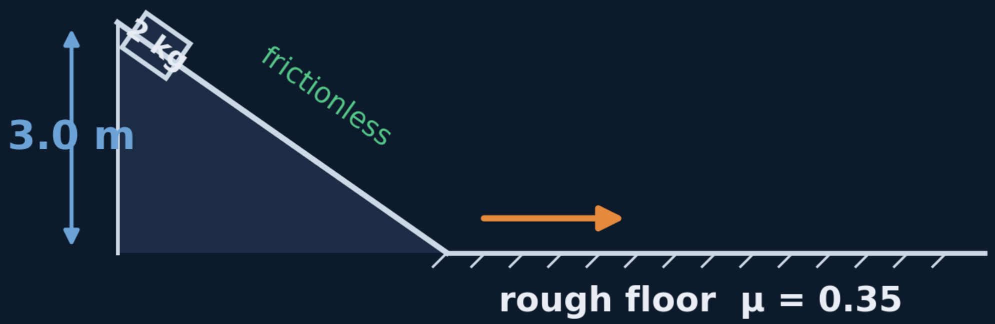
5. A 3.0 kg block rests on a frictionless table, connected by a rope over a pulley at the edge to a 2.0 kg mass hanging in the air. The system is released from rest. Find the speed of the blocks after the hanging mass drops 1.0 m. (Both blocks move at the same speed — that is what sets the kinetic energy term.)



Multistep Conservation of Energy

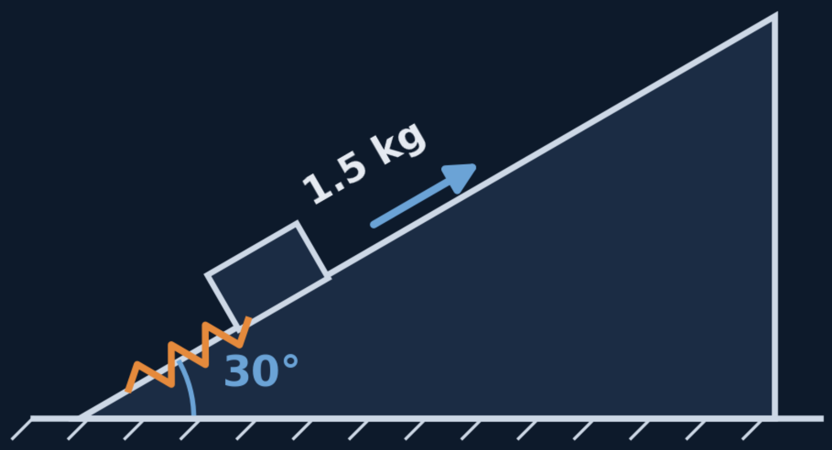
MEDIUM

6. A 2.0 kg block is released from rest at the top of a frictionless ramp 3.0 m high, then slides onto a rough floor (μ = 0.35) and stops. How far does it travel across the floor?



HARD

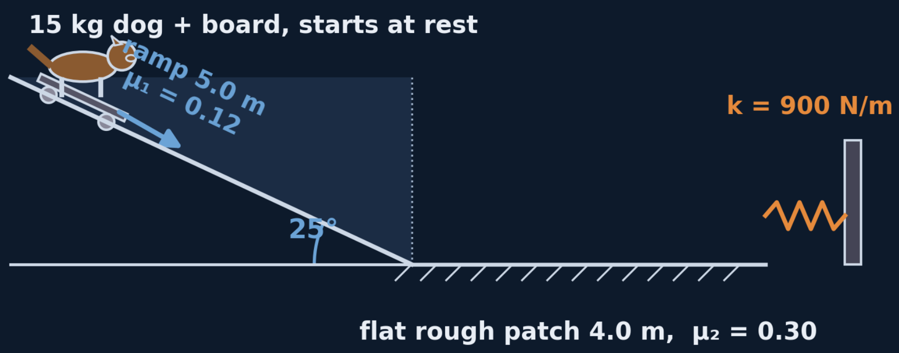
7. A 1.5 kg cart is pressed against a spring (k = 1800 N/m) compressed 0.25 m at the base of a 30° incline with μ = 0.18. Released, it launches up the incline. How far along the incline does it travel before it momentarily stops?



k = 1800 N/m, compressed 0.25 m, μ = 0.18

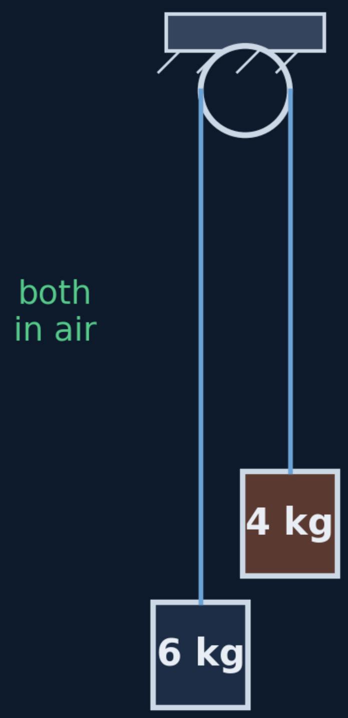
DIABOLICAL

8. A 15 kg dog on a skateboard (one 15 kg object, massless board) starts from rest at the top of a 5.0 m ramp inclined at 25° with $\mu_1 = 0.12$. At the bottom it rolls onto a flat rough patch 4.0 m long with $\mu_2 = 0.30$, then strikes a spring ($k = 900$ N/m). Find the maximum compression of the spring.



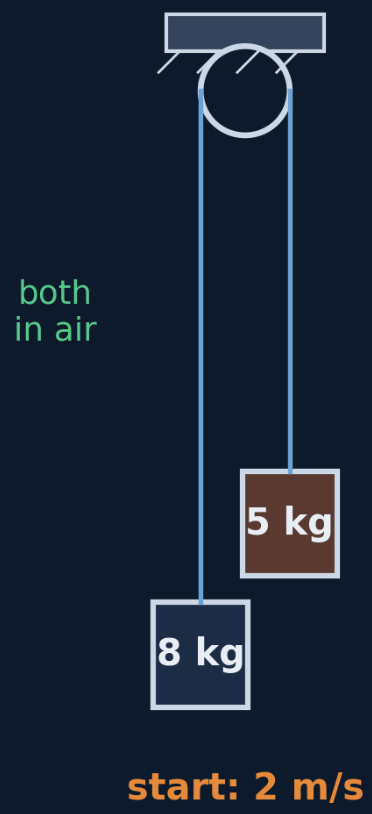
MEDIUM

9. Two masses hang from a light frictionless pulley: 6.0 kg and 4.0 kg, both in the air. Released from rest, find (a) the speed after the 6.0 kg mass descends 1.5 m, and (b) the kinetic energy of the system at that instant.



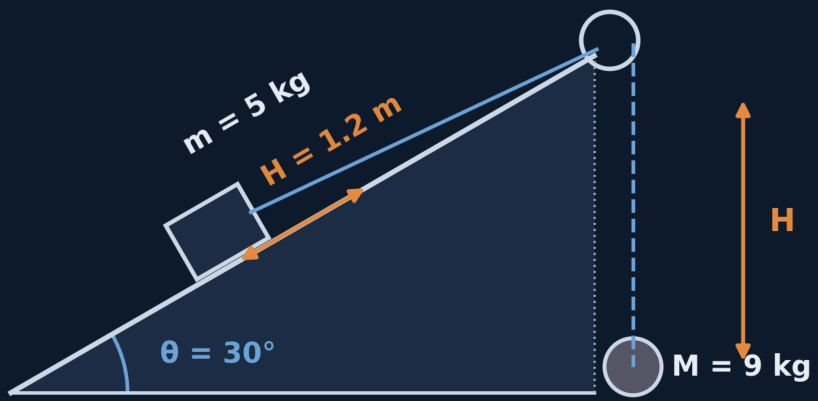
HARD

10. The Atwood machine shown (8.0 kg and 5.0 kg, both in the air) is already moving at 2.0 m/s with the 8.0 kg mass descending. Find the speed after the 8.0 kg mass descends a further 2.0 m.



DIABOLICAL

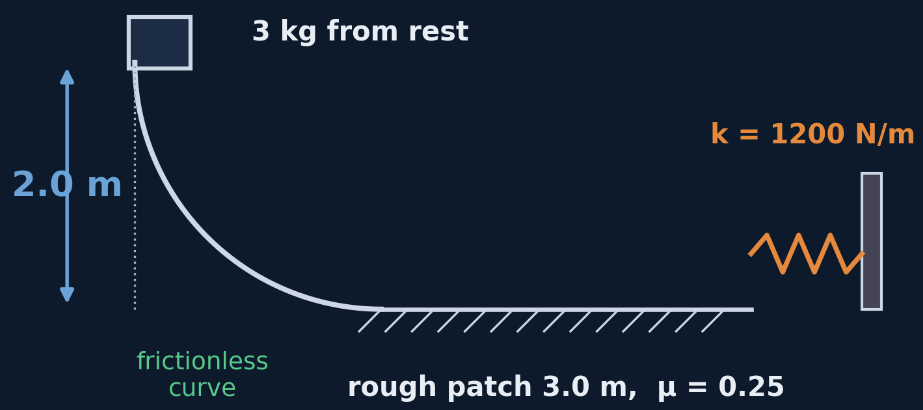
11. A 5.0 kg block on a 30° incline ($\mu = 0.20$) is joined by a rope over a pulley at the apex to a 9.0 kg mass hanging freely, as shown. Released from rest, find the speed after the hanging mass descends H = 1.2 m. (The block slides the same distance H up the incline, rising $H \cdot \sin 30°$, and friction acts over the full distance H.)



μ = 0.20 on incline, released from rest

DIABOLICAL

12. A 3.0 kg block slides from rest down the frictionless quarter-circle ramp shown (radius 2.0 m). It crosses a rough patch 3.0 m long (μ = 0.25), compresses a spring (k = 1200 N/m), is pushed back, recrosses the same patch, and climbs the ramp again. Find (a) the max spring compression, (b) the speed when it returns to the base of the ramp, and (c) how high it rises the second time.



Answer Key — Standard 4.2

1 — Bar charts + speed at the bottom (frictionless)

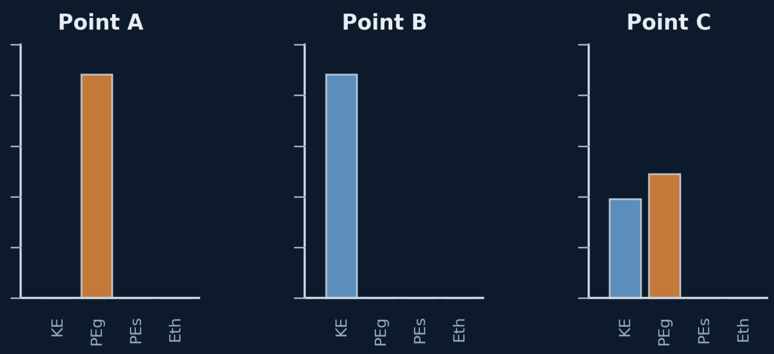
Total energy stays 8.82 J the whole time (no friction, so Eth = 0 always).

A: all gravitational — PEg = mgh = 0.5 × 9.8 × 1.8 = 8.82 J, KE = 0.

B: all kinetic — KE = 8.82 J, PEg = 0. C: PEg = 0.5 × 9.8 × 1.0 = 4.9 J, KE = 8.82 − 4.9 = 3.92 J.

(b) At B: $\frac{1}{2}mv^2 = 8.82 \rightarrow v = \sqrt{2\times8.82/0.5}$ = **5.94 m/s**.

Your bars should look like this:



Answer: Speed at B = 5.94 m/s; charts as shown (Eth stays zero throughout)

Common error: Drawing any thermal energy on a frictionless track. With no friction, energy only shuffles between KE and PEg.

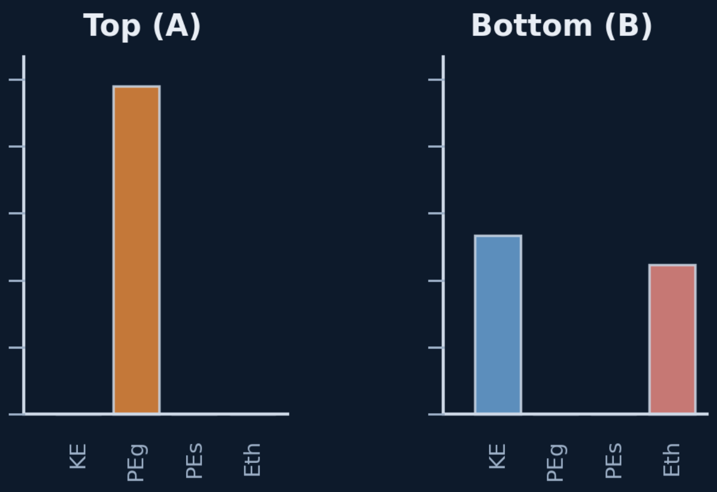
2 — Bar charts with friction

A: PEg = mgh = 2.0 × 9.8 × 1.5 = 29.4 J, everything else 0.

B: KE = $\frac{1}{2}mv^2$ = ½ × 2.0 × 4.0^2 = 16.0 J, PEg = 0.

Energy is conserved, so the missing energy is now thermal: Eth = 29.4 − 16.0 = **13.4 J**.

Your bars should look like this:



Answer: Thermal energy generated = 13.4 J (total stays 29.4 J)

Common error: Saying 13.4 J was "lost." It was not destroyed — it became thermal energy, which the bar chart must show.

3 — Reading a bar chart

Total energy is conserved at 60 J.

(a) At Point 2: KE = 60 − PEg − Eth = 60 − 20 − 10 = **30 J**.

(b) $\frac{1}{2}mv^2$ = 30 → v = √(2×30/3.0) = √20 = **4.47 m/s**.

Answer: (a) 30 J (b) 4.47 m/s

Common error: Ignoring the thermal bar when balancing energy. Thermal energy is part of the total and must be subtracted.

4 — Same height, same speed, different time

Energy depends only on the drop, not the path: $\frac{1}{2}mv^2 = mgh \rightarrow v = \sqrt{2gh}$.

(a) Both: $v = \sqrt{2 \times 9.8 \times 2.0} = \sqrt{39.2}$ = **6.26 m/s**.

(b) Yes — the speeds are equal, because both blocks fall the same height.

(c) No. Acceleration along a frictionless ramp is $g \cdot \sin\theta$, larger on the steep ramp, and the steep ramp is also shorter. Working the times: steep ≈ 0.74 s, gentle ≈ 1.87 s.

So they arrive at the same speed but the steep block gets there first.

Answer: (a) both 6.26 m/s (b) equal (c) not the same time — steep ≈ 0.74 s, gentle ≈ 1.87 s

Common error: Assuming equal final speeds mean equal arrival times. Energy fixes the speed; the ramp's steepness fixes the time.

5 — Connected blocks share one speed

Only the hanging mass changes height; both blocks move at the same speed v, so $KE = \frac{1}{2}(m_1+m_2)v^2$.

PE lost by the hanging mass = shared KE: $m_{hang}\, g\, d = \frac{1}{2}(m_1+m_2)v^2$.

$2.0 \times 9.8 \times 1.0 = \frac{1}{2} \times (3.0+2.0) \times v^2 \rightarrow 19.6 = 2.5\, v^2 \rightarrow v^2 = 7.84 \rightarrow v =$ **2.8 m/s**.

Answer: 2.8 m/s

Common error: Putting only the hanging mass in the kinetic-energy term. The table block moves too, at the same speed, so it carries KE as well.

6 — Energy down a ramp, lost on a floor

KE at the bottom = $mgh = 2.0 \times 9.8 \times 3.0 = 58.8$ J.

Friction removes it: $\mu mg \cdot d = 58.8 \rightarrow 6.86\, d = 58.8 \rightarrow d =$ **8.57 m**.

Answer: 8.57 m

Common error: Using the 3.0 m height as the floor distance. Height sets the energy; the floor distance is what you solve for.

7 — Spring launch up a rough incline

Spring energy = $\frac{1}{2}kx^2 = \frac{1}{2} \times 1800 \times 0.25^2 = 56.25$ J.

Up the incline, gravity and friction both oppose over distance d:

$mg \sin 30° \cdot d + \mu mg \cos 30° \cdot d = (7.35 + 2.29)\, d = 9.64\, d = 56.25$.

d = **5.83 m** along the incline.

Answer: 5.83 m

Common error: Swapping sin and cos. Gravity along the incline uses sin(angle); friction uses cos(angle).

8 — Dog on a skateboard: two friction coefficients

Ramp: height = 5.0 × sin25° = 2.113 m, PE = 15 × 9.8 × 2.113 = 310.6 J.

Ramp friction = μ_1 mg cos25° × 5.0 = 79.9 J → KE at base = 230.7 J.

Flat patch friction = μ_2 mg × 4.0 = 176.4 J (no cos on level ground) → KE at spring = 54.3 J.

½kx^2 = 54.3 → 450 x^2 = 54.3 → x = **0.347 m**.

Answer: 0.347 m

Common error: Using one friction coefficient for both surfaces, or applying cos25° to the flat-ground friction. Each surface has its own μ.

9 — Atwood machine, both in the air

$(m_1 - m_2)\, g\, d = ½(m_1 + m_2)\, v^2$.

(6.0−4.0) × 9.8 × 1.5 = ½ × 10 × v^2 → 29.4 = 5 v^2 → v = **2.42 m/s**.

(b) KE = ½ × 10 × 5.88 = **29.4 J**.

Answer: (a) 2.42 m/s (b) 29.4 J

Common error: Putting only the falling mass in the KE term. Both masses move at the same speed, so KE uses the sum.

10 — Atwood already moving

Include the starting KE: $½(m_1+m_2)v_i^2 + (m_1-m_2)g\,d = ½(m_1+m_2)v_f^2$.

½ × 13 × 4.0 + 3 × 9.8 × 2.0 = 6.5 v_f^2 → 26 + 58.8 = 6.5 v_f^2.

v_f^2 = 13.05 → v_f = **3.61 m/s**.

Answer: 3.61 m/s

Common error: Starting from rest and dropping the initial kinetic-energy term.

11 — Incline block joined to a hanging mass

MgH = mg(H sin30°) + ½(M+m)v^2 + fH, with f = μ mg cos30°.

MgH = 9.0 × 9.8 × 1.2 = 105.84 J; block PE gain = 5.0 × 9.8 × 1.2 × 0.5 = 29.4 J.

f = 0.20 × 5.0 × 9.8 × 0.8660 = 8.487 N → heat = fH = 10.18 J.

½(14)v^2 = 105.84 − 29.4 − 10.18 = 66.26 → v = **3.08 m/s**.

Answer: 3.08 m/s

Common error: Forgetting the KE term uses (M+m), or using H·sin30° as the friction distance. Friction acts over the full distance H the block slides.

12 — Full round trip: ramp, rough patch twice, spring

Down the ramp: KE at base = mgh = $3.0 \times 9.8 \times 2.0 = 58.8$ J.

First crossing: loss = $\mu mg \cdot d = 0.25 \times 3.0 \times 9.8 \times 3.0 = 22.05$ J → KE at spring = 36.75 J.

(a) $\frac{1}{2}kx^2 = 36.75 \rightarrow x =$ **0.247 m**.

Spring returns 36.75 J; second crossing loses 22.05 J again → KE at base = 14.70 J.

(b) $v = \sqrt{(2 \times 14.70/3.0)}$ = **3.13 m/s**. (c) mgh = 14.70 → h = **0.50 m**.

Answer: (a) 0.247 m (b) 3.13 m/s (c) 0.50 m

Common error: Charging friction only once. The block pays the toll on every crossing, so the return trip loses energy again.

AUX — Free Physics Resources · auxlearning.com · g = 9.8 m/s^2 throughout