Unit 6.1 — Forces and Circular Motion

Coulomb's Law · Universal Gravitation · Centripetal Force

Hardest Practice Set · 22 problems · Full answer key begins after Problem 22

Constants: $k = 8.99\times10^{9}$ N·m^2/C^2 | $G = 6.674\times10^{-11}$ N·m^2/kg^2 | $g = 9.8$ m/s^2 | $M_E = 5.97\times10^{24}$ kg | $R_E = 6.37\times10^{6}$ m

$F = k|q_1||q_2|/r^2$ $F = GMm/r^2$ $a_c = v^2/r = 4\pi^2 r/T^2$ $F_{net} = mv^2/r$

SECTION A — Coulomb's Law: Magnitude, Sign, and Scaling

1. Two charges of +6.0 μC and −4.0 μC are separated by 0.30 m. Find the magnitude of the electrostatic force between them and state whether it is attractive or repulsive.

2. Two −5.0 nC charges experience a force of 450 N. How far apart are they?

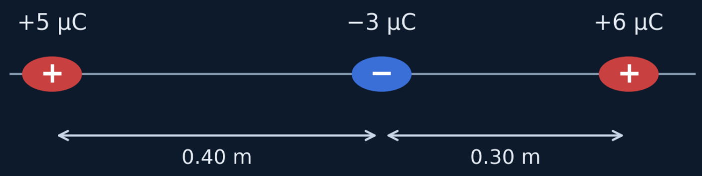
3. A +9.0 nC charge and a +4.0 pC charge are separated by 3.0 μm. Find the force between them.

Watch the prefixes: nC = 10^{-9} C, pC = 10^{-12} C, μm = 10^{-6} m.

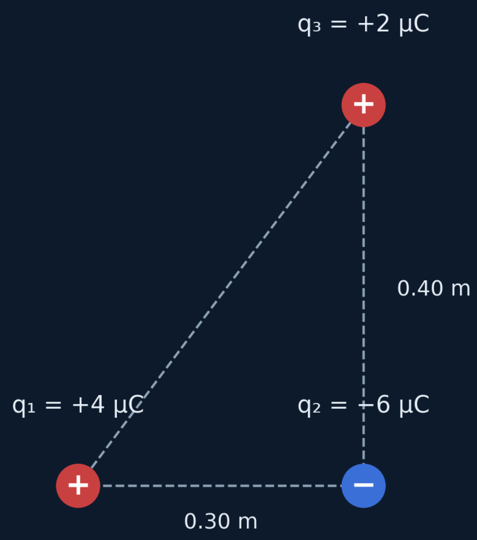
4. A pair of charges experiences a force of 6.4×10^{5} N. You quadruple one of the charges **and** triple the separation. What is the new force?

SECTION B — Net Force from Multiple Charges

5. Three charges lie on a straight line as shown below. Find the magnitude and direction of the net electrostatic force on the −3 μC charge.

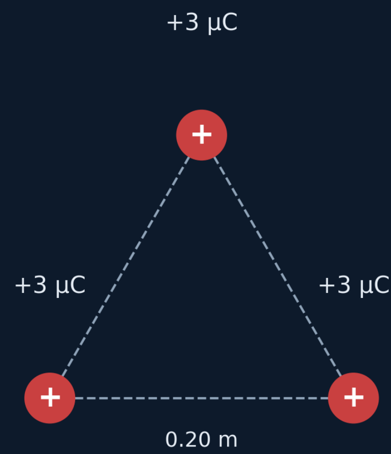


6. Three charges are arranged as shown. Find the magnitude and direction of the net force on q_3. Draw both force vectors on the diagram before you compute anything.



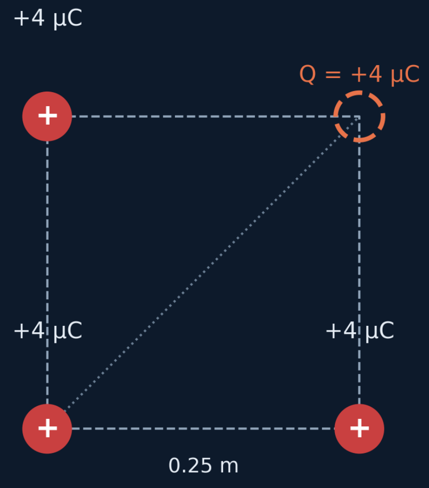
7. Three identical +3.0 μC charges sit at the corners of an equilateral triangle of side 0.20 m. Find the magnitude of the net force on any one of them.

Use symmetry. The two forces on any corner charge are separated by 60°.



8. A charge +2q sits at x = 0 and a charge +8q sits at x = 1.20 m. Find the location on the line between them where a small test charge would feel zero net force.

9. Three +4.0 μC charges sit at three corners of a square of side 0.25 m. A fourth charge Q = +4.0 μC is placed at the empty corner. Find the magnitude and direction of the net force on Q.



SECTION C — Newton’s Law of Universal Gravitation

10. Two 5000 kg trucks are parked 3.0 m apart. Find the gravitational force between them.

11. Mars has a surface gravitational field strength of 3.7 m/s^2 and a radius of 3.39×10^6 m. Find the mass of Mars.

12. A satellite orbits at an altitude of 400 km above Earth's surface. Find the gravitational field strength g at that altitude, and state what fraction of the surface value it represents.

Remember that r is measured from the CENTER of the Earth, not from the surface.

SECTION D — Centripetal Force and Acceleration

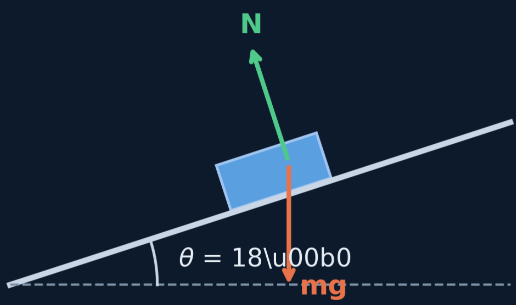
13. A 1500 kg car rounds a flat curve of radius 80.0 m at 25.0 m/s. Find the centripetal acceleration and the friction force required to keep the car on the curve.

14. A ball on a 2.00 m string completes one full revolution every 1.50 s. Find its centripetal acceleration using $a_c = 4\pi^2 r/T^2$.

15. A 0.25 kg ball is swung in a horizontal circle on a 0.90 m string that snaps at 80.0 N of tension. Find the maximum speed the ball can reach before the string breaks.

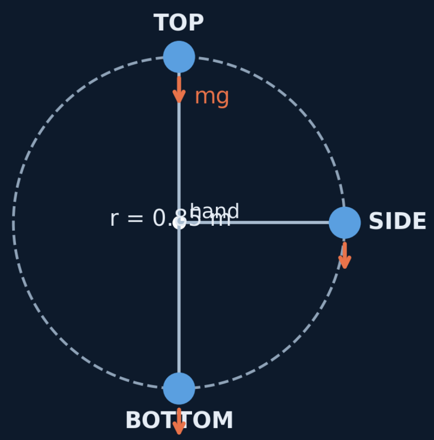
SECTION E — The Hardest Tier: Mixed and Multi-Step

16. Banked curve. A curve of radius 120 m is banked at 18.0°. A 1100 kg car takes the curve at exactly the speed for which **no friction is needed**. (a) Find that speed. (b) Find the normal force on the car. (c) Explain in one sentence which component of the normal force supplies the centripetal force.



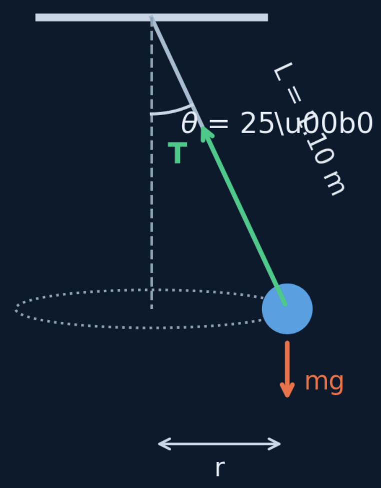
17. Vertical circle. A 0.60 kg ball is swung in a vertical circle of radius 0.85 m at a constant speed of 6.00 m/s. Find the string tension at the **bottom**, at the **top**, and at the **side** (the horizontal position). Then find the minimum speed at the top for which the string stays taut.

At each position, decide whether gravity points toward the center, away from it, or perpendicular to the radius.



18. Conical pendulum. A 0.40 kg ball on a 1.10 m string swings in a horizontal circle, with the string making a constant 25.0° angle with the vertical. Find (a) the tension in the string, (b) the radius of the circle, (c) the speed of the ball, and (d) the period of the motion.

The vertical component of tension balances gravity. The horizontal component supplies the centripetal force.



19. The space yo-yo. A gorilla floating in space swings a ginormous yo-yo of mass 50.0 kg on a massless string of radius R = 3.00×10^6 m, in a vertical circle at a constant speed of 4.00×10^3 m/s. The gorilla is positioned so that the **bottom** of the circle just grazes Earth's surface, which means the **top** of the circle sits at r = 1.237×10^7 m from Earth's center.

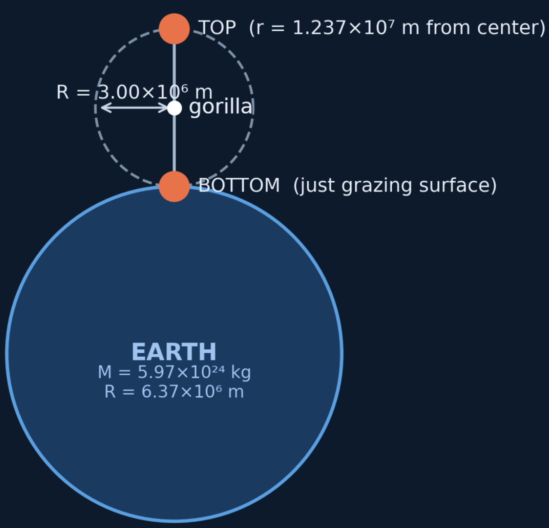
(a) Find the string tension at the **bottom**. At this position the yo-yo is essentially at Earth's surface, so you may use the weight mg.

(b) Find the gravitational field strength at the **top** of the circle using g = GM/r^2.

(c) Find the string tension at the **top**. You cannot use mg here — gravity has weakened substantially.

(d) Explain why the tension at the top is so much smaller than at the bottom, naming **both** reasons.

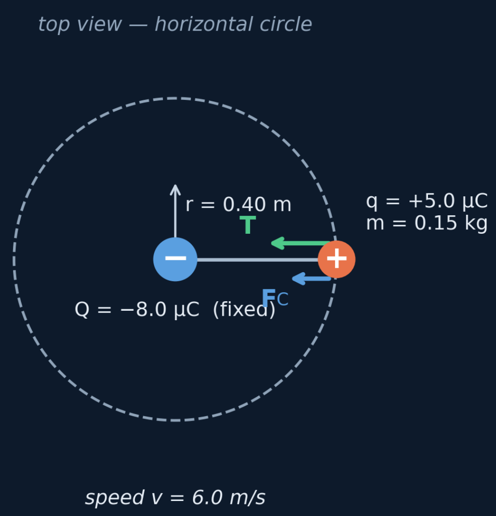
At the bottom, gravity points AWAY from the center of the circle. At the top, gravity points TOWARD it.



yo-yo mass m = 50.0 kg, constant speed v = 4.00×10^3 m/s

20. Coulomb shares the load. A 0.15 kg bead carrying a charge of +5.0 μC is tied by a string to a fixed −8.0 μC charge and swings around it in a horizontal circle of radius 0.40 m at 6.0 m/s. Both the string tension **and** the Coulomb attraction pull the bead inward. (a) Find the centripetal force required. (b) Find the Coulomb force. (c) Find the string tension. (d) Find the speed at which the string would go completely slack.

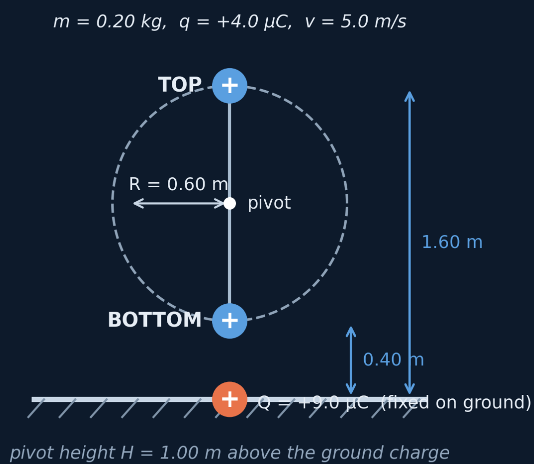
Coulomb does not have to supply the WHOLE centripetal force. Here it supplies part of it, and the string covers the rest.



21. Charged vertical circle. A 0.20 kg ball carrying +4.0 μC is swung in a vertical circle of radius 0.60 m at a constant 5.0 m/s. The pivot sits 1.00 m directly above a fixed +9.0 μC charge on the ground, so the ball passes 0.40 m above that charge at the bottom and 1.60 m above it at the top. The ground charge repels the ball, always pushing it straight up.

(a) Find the Coulomb force at the bottom and at the top. (b) Find the string tension at the **bottom**. (c) Find the string tension at the **top**. (d) The Coulomb force does two different things at these two positions. Explain what changes about its **direction relative to the center**, and separately what changes about its **magnitude**.

At the bottom the center of the circle is above the ball. At the top the center is below it. An upward push is not always an inward push.



22. Charged orbit. An electron ($m = 9.11\times10^{-31}$ kg, $q = 1.60\times10^{-19}$ C) orbits a proton in a circle of radius 5.29×10^{-11} m. The Coulomb attraction supplies the entire centripetal force. Set $kq^2/r^2 = mv^2/r$ and find (a) the electron's orbital speed and (b) its orbital period.

Answer Key

Unit 6.1 — Forces and Circular Motion · Full worked solutions

1. Coulomb's law, two charges

$F = k|q_1||q_2|/r^2$

$F = (8.99\times10^{9})(6.0\times10^{-6})(4.0\times10^{-6})/(0.30)^2$

$F = (8.99\times10^{9})(2.4\times10^{-11})/0.090$

→ F = 2.40 N, attractive (opposite signs)

Common error: Putting the minus sign into the formula. Use |q| for magnitude; the signs only set the direction.

2. Solve for separation

$F = kq^2/r^2 \rightarrow r = \sqrt{(kq^2/F)}$

$r = \sqrt{[(8.99\times10^{9})(5.0\times10^{-9})^2/450]}$

$r = \sqrt{(4.99\times10^{-10})}$

→ $r = 2.23\times10^{-5}$ m

Common error: Forgetting to take the square root at the end, or squaring the nC prefix incorrectly.

3. Mixed SI prefixes

$q_1 = 9.0\times10^{-9}$ C, $q_2 = 4.0\times10^{-12}$ C, $r = 3.0\times10^{-6}$ m

$F = (8.99\times10^{9})(9.0\times10^{-9})(4.0\times10^{-12})/(3.0\times10^{-6})^2$

$F = (3.236\times10^{-10})/(9.0\times10^{-12})$

→ F = 36.0 N, repulsive (both positive)

Common error: Treating pC as 10^{-9}. A single wrong prefix changes the answer by a factor of 1000.

4. Proportional scaling

$F \propto q_1q_2/r^2$

charge factor = 4; distance factor = 3 → divides by $3^2 = 9$

net factor = 4/9

$F' = (6.4\times10^{5})(4/9)$

→ $F' = 2.84\times10^{5}$ N

Common error: Dividing by 3 instead of 9. Distance always enters squared.

5. Net force, collinear charges

From +5 μC (attracts the −3 μC to the LEFT):

$F_1 = k(5.0\times10^{-6})(3.0\times10^{-6})/(0.40)^2 = 0.843$ N (−x)

From +6 μC (attracts the −3 μC to the RIGHT):

$F_2 = k(6.0\times10^{-6})(3.0\times10^{-6})/(0.30)^2 = 1.798$ N (+x)

$\Sigma F = 1.798 - 0.843$

→ ΣF = 0.955 N, directed toward the +6 μC charge (+x)

Common error: Adding the magnitudes. The two forces oppose each other — subtract.

6. Net force on a right triangle

$r_{13} = \sqrt{(0.30^2 + 0.40^2)} = 0.50$ m

$F_{13} = k(4.0\times10^{-6})(2.0\times10^{-6})/(0.50)^2 = 0.288$ N, repulsive

unit vector (0.30, 0.40)/0.50 = (0.60, 0.80)

$F_{13} = (0.173, 0.230)$ N

$F_{23} = k(6.0\times10^{-6})(2.0\times10^{-6})/(0.40)^2 = 0.674$ N, attractive → (0, −0.674) N

$\Sigma F = (0.173, -0.444)$ N

$|\Sigma F| = \sqrt{(0.173^2 + 0.444^2)}$

→ |ΣF| = 0.476 N, at 68.8° below the +x axis

Common error: Using r = 0.40 m for the force from q_1. The distance is the hypotenuse, 0.50 m.

7. Equilateral triangle, symmetry

Each pairwise force: $F = k(3.0\times10^{-6})^2/(0.20)^2 = 2.023$ N

The two forces on the corner charge are 60° apart.

Net = 2F cos(30°) = 2(2.023)(0.866)

→ F_{net} = 3.50 N, directed outward along the bisector (away from the triangle’s center)

Common error: Using cos(60°). The bisector splits the 60° angle, so each force makes 30° with the resultant.

8. Null point between like charges

$k(2q)/r_1^2 = k(8q)/r_2^2$

$r_2^2/r_1^2 = 4 \rightarrow r_2 = 2r_1$

$r_1 + r_2 = 1.20 \rightarrow 3r_1 = 1.20$

→ r_1 = 0.40 m from the +2q charge (0.80 m from the +8q)

Common error: Placing the null point at the midpoint. It always sits closer to the SMALLER charge.

9. Square, three-charge superposition

Two adjacent charges, each at r = 0.25 m:

$F_{adj} = k(4.0\times10^{-6})^2/(0.25)^2 = 2.301$ N each, along +x and +y

Diagonal charge at $r = 0.25\sqrt{2} = 0.354$ m:

$F_{diag} = k(4.0\times10^{-6})^2/(0.354)^2 = 1.151$ N, along (0.707, 0.707)

$\Sigma F_x = 2.301 + 0.814 = 3.115$ N; $\Sigma F_y = 3.115$ N

$|\Sigma F| = \sqrt{2}\,(3.115)$

→ |ΣF| = 4.41 N, directed outward along the diagonal at 45°

Common error: Using r = 0.25 m for the diagonal charge. The diagonal of a square is side×√2.

10. Gravitation between everyday masses

$F = GMm/r^2$

$F = (6.674\times10^{-11})(5000)(5000)/(3.0)^2$

→ $F = 1.85\times10^{-4}$ N

Common error: Expecting a large number. Gravity between ordinary objects is almost undetectable.

11. Solve for planetary mass

$g = GM/R^2 \rightarrow M = gR^2/G$

$M = (3.7)(3.39\times10^6)^2/(6.674\times10^{-11})$

$M = (3.7)(1.149\times10^{13})/(6.674\times10^{-11})$

→ $M = 6.37\times10^{23}$ kg

Common error: Confusing G with g. G is the universal constant; g is the local field strength.

12. Field strength at altitude

$r = R_E + h = 6.37\times10^6 + 4.00\times10^5 = 6.77\times10^6$ m

$g = GM_E/r^2 = (6.674\times10^{-11})(5.97\times10^{24})/(6.77\times10^6)^2$

fraction = 8.69/9.8

→ $g = 8.69$ m/s^2, about 89% of the surface value

Common error: Using r = 400 km. Distance is measured from Earth’s CENTER, so add the radius.

13. Flat curve

$a_c = v^2/r = (25.0)^2/80.0$

$F = ma_c = 1500(7.81)$

→ $a_c = 7.81$ m/s^2; $F_{friction} = 1.17\times10^4$ N

Common error: Adding an outward force to the free-body diagram. Friction points INWARD — it is the only horizontal force.

14. Period form of centripetal acceleration

$a_c = 4\pi^2 r/T^2$

$a_c = 4\pi^2(2.00)/(1.50)^2$

$a_c = 78.96/2.25$

→ $a_c = 35.1$ m/s^2

Common error: Forgetting to square T. A small period produces an enormous acceleration.

15. Breaking tension

$T = mv^2/r \rightarrow v = \sqrt{(Tr/m)}$

$v = \sqrt{[(80.0)(0.90)/0.25]}$

$v = \sqrt{288}$

→ $v_{max} = 17.0$ m/s

Common error: Dividing by r instead of multiplying. Rearrange the equation symbolically before substituting.

16. Banked curve, frictionless

Vertical: $N \cos\theta = mg$

Horizontal: $N \sin\theta = mv^2/r$

Divide: $\tan\theta = v^2/(rg)$

(a) $v = \sqrt{(rg \tan\theta)} = \sqrt{[(120)(9.8)(\tan 18.0°)]} = \sqrt{382.1}$

$v = 19.5$ m/s

(b) $N = mg/\cos\theta = (1100)(9.8)/\cos 18.0° = 10780/0.951$

$N = 1.13\times10^4$ N

(c) The HORIZONTAL component $N \sin\theta$ points toward the center of the curve and supplies the entire centripetal force. The vertical component $N \cos\theta$ balances gravity.

→ $v = 19.5$ m/s; $N = 1.13\times10^4$ N

Common error: Using N = mg. On a bank the normal force is LARGER than the weight, because only its vertical component balances gravity.

17. Vertical circle, all three positions

$mv^2/r = (0.60)(6.00)^2/0.85 = 25.41$ N (the centripetal requirement, same everywhere)

$mg = (0.60)(9.8) = 5.88$ N

Bottom: gravity points AWAY from center → $T - mg = mv^2/r$

$T = 25.41 + 5.88 = 31.3$ N

Top: gravity points TOWARD center → $T + mg = mv^2/r$

$T = 25.41 - 5.88 = 19.5$ N

Side: gravity is perpendicular to the radius, contributes nothing → $T = mv^2/r = 25.4$ N

Minimum speed at top: set $T = 0$ → $mg = mv^2/r$ → $v = \sqrt{(gr)}$

$v = \sqrt{[(9.8)(0.85)]} = 2.89$ m/s

→ $T_{bottom} = 31.3$ N; $T_{top} = 19.5$ N; $T_{side} = 25.4$ N; $v_{min} = 2.89$ m/s

Common error: Using $T - mg = mv^2/r$ at the top. Gravity HELPS pull inward there, so it adds to the tension on the left side, not subtracts.

18. Conical pendulum

Vertical: $T \cos\theta = mg$

(a) $T = mg/\cos\theta = (0.40)(9.8)/\cos 25.0° = 3.92/0.906$

$T = 4.33$ N

(b) $r = L \sin\theta = (1.10)(\sin 25.0°)$

$r = 0.465$ m

Horizontal: $T \sin\theta = mv^2/r$

(c) $v = \sqrt{(T r \sin\theta/m)} = \sqrt{[(4.33)(0.465)(0.4226)/0.40]} = \sqrt{2.125}$

$v = 1.46$ m/s

(d) $T_{period} = 2\pi r/v = 2\pi(0.465)/1.46$

→ $T = 4.33$ N; $r = 0.465$ m; $v = 1.46$ m/s; period = 2.00 s

Common error: Using r = L. The radius is the HORIZONTAL distance, L sinθ — not the string length.

19. The space yo-yo — gravitation meets centripetal force

The centripetal requirement is the same at every point:

$mv^2/R = (50.0)(4.00\times10^3)^2/(3.00\times10^6) = 8.00\times10^8/3.00\times10^6 = 266.7$ N

(a) Bottom. Yo-yo grazes the surface, so gravity = mg = (50.0)(9.8) = 490 N, pointing AWAY from the circle's center (down toward Earth).

$T - mg = mv^2/R$

$T = 266.7 + 490 = 757$ N

(b) Top. $g = GM_E/r^2 = (6.674\times10^{-11})(5.97\times10^{24})/(1.237\times10^7)^2$

$g = 3.984\times10^{14}/1.530\times10^{14} = 2.60\ \text{m/s}^2$

(c) Top tension. Gravity now points TOWARD the circle's center (down toward Earth, which is below the top of the circle).

$F_{grav} = mg_{top} = (50.0)(2.60) = 130.2$ N

$T + F_{grav} = mv^2/R$

$T = 266.7 - 130.2 = 137$ N

(d) Two reasons stack. First, gravity has flipped role: at the bottom it opposes the tension (so tension must exceed the centripetal requirement), while at the top it points inward and does part of the job for you. Second, gravity itself has weakened from 9.8 to 2.60 m/s^2 because r has nearly doubled, and the field falls off as $1/r^2$.

→ T_{bottom} = 757 N; g_{top} = 2.60 m/s^2; T_{top} = 137 N

Common error: Using mg = 490 N at the top as well. At $r = 1.237\times10^7$ m the field has dropped to about a quarter of its surface value — you must recompute it with GM/r^2.

20. Coulomb shares the centripetal load

(a) Centripetal requirement:

$mv^2/r = (0.15)(6.0)^2/0.40 = 5.40/0.40 = 13.5$ N

(b) Coulomb force (opposite signs → attractive → inward):

$F_C = k(8.0\times10^{-6})(5.0\times10^{-6})/(0.40)^2 = 0.3596/0.16 = 2.25$ N

(c) Both forces point inward, so they ADD to meet the requirement:

$T + F_C = mv^2/r$

$T = 13.5 - 2.25 = 11.25$ N

(d) String goes slack when T = 0, so Coulomb alone must supply everything:

$F_C = mv^2/r \rightarrow v = \sqrt{(F_C\, r/m)} = \sqrt{[(2.2475)(0.40)/0.15]} = \sqrt{5.993}$

→ F_C = 2.25 N; T = 11.25 N; slack at v = 2.45 m/s

Common error: Subtracting the Coulomb force from the tension. Both forces point inward here, so together they equal mv^2/r — the tension only has to cover the shortfall.

21. Charged vertical circle — role flip and magnitude change

Centripetal requirement (same at every point):

$mv^2/R = (0.20)(5.0)^2/0.60 = 5.00/0.60 = 8.333$ N

$mg = (0.20)(9.8) = 1.96$ N

(a) Coulomb force, like charges → repulsive → always pushes the ball UP:

$F_{C,bottom} = k(9.0\times10^{-6})(4.0\times10^{-6})/(0.40)^2 = 0.3236/0.16 = 2.023$ N

$F_{C,top} = k(9.0\times10^{-6})(4.0\times10^{-6})/(1.60)^2 = 0.3236/2.56 = 0.126$ N

(b) Bottom. Center is ABOVE the ball, so inward = up.

T points up (inward), F_C points up (inward), gravity points down (outward):

$T + F_C - mg = mv^2/R$

$T = 8.333 - 2.023 + 1.96 = 8.27$ N

(c) Top. Center is BELOW the ball, so inward = down.

T points down (inward), gravity points down (inward), F_C points up (OUTWARD):

$T + mg - F_C = mv^2/R$

$T = 8.333 - 1.96 + 0.126 = 6.50$ N

(d) Direction: the Coulomb push is always upward, but "up" is inward at the bottom and outward at the top, so it switches from helping the tension to fighting it. Magnitude: the distance to the ground charge grows from 0.40 m to 1.60 m, a factor of 4, and force falls as $1/r^2$, so it drops by a factor of 16 — from 2.023 N to 0.126 N.

→ $F_{C,bot}$ = 2.02 N, $F_{C,top}$ = 0.126 N; T_{bottom} = 8.27 N; T_{top} = 6.50 N

Common error: Assuming the Coulomb force is inward at both positions because it always points up. Inward is defined by the CENTER of the circle, and the center moves from above the ball to below it.

22. Electron orbit — Coulomb supplies the centripetal force

$kq^2/r^2 = mv^2/r \rightarrow v = \sqrt{(kq^2/(mr))}$

(a) $v = \sqrt{[(8.99\times10^9)(1.60\times10^{-19})^2/((9.11\times10^{-31})(5.29\times10^{-11}))]}$

$v = \sqrt{[(2.301\times10^{-28})/(4.819\times10^{-41})]} = \sqrt{(4.775\times10^{12})}$

$v = 2.19\times10^6$ m/s

(b) $T = 2\pi r/v = 2\pi(5.29\times10^{-11})/(2.19\times10^6)$

→ $v = 2.19\times10^6$ m/s; period $T = 1.52\times10^{-16}$ s

Common error: Cancelling r incorrectly. One power of r survives on the left, giving $v^2 = kq^2/(mr)$, not $kq^2/(mr^2)$.