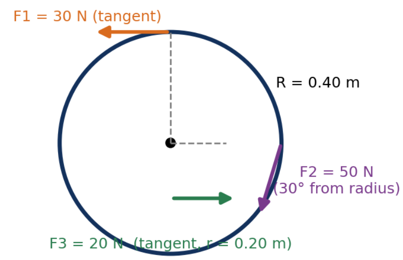
Standard 3.1 — Torque and Center of Mass

Hard Practice Set

Covers torque, rotational equilibrium, centre of mass, and levers. $g = 9.8\ m/s^2$. Simple calculator only. Show all work and carry units. These problems are built at the hardest tested level for this standard.

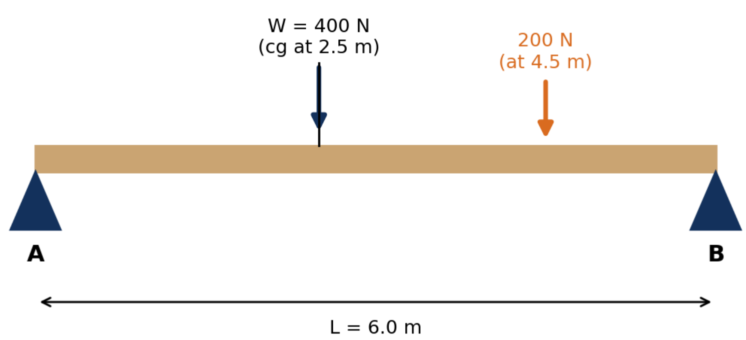
Problem 1 [Torque]

A wheel of radius R = 0.40 m turns about its centre. Three forces act in its plane (see diagram): F_1 = 30 N applied tangentially at the rim (tends to turn it counterclockwise); F_2 = 50 N applied at the rim but directed 30° away from the radius; and F_3 = 20 N applied tangentially at a radius of 0.20 m, turning it clockwise. Find the magnitude and direction of the net torque.



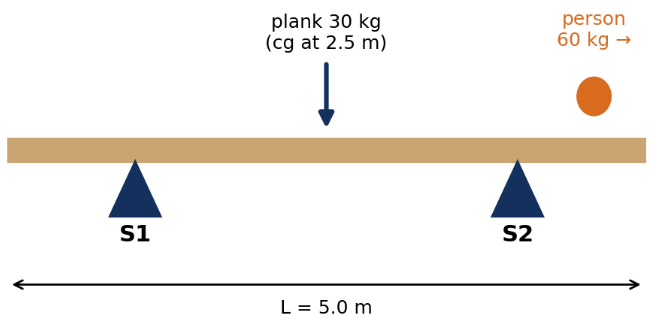
Problem 2 [Rotational equilibrium]

A non-uniform beam 6.0 m long weighs 400 N, with its centre of gravity 2.5 m from the left end A. It rests on a support at each end (A and B). A 200 N load sits 4.5 m from the left end. Find the upward force each support provides.



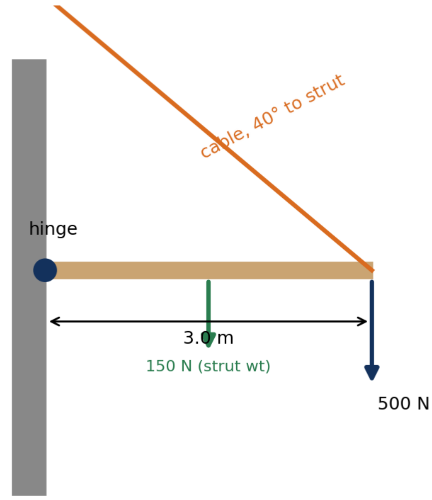
Problem 3 [Rotational equilibrium]

A uniform plank 5.0 m long has mass 30 kg. It rests on two supports: S1 at 1.0 m and S2 at 4.0 m from the left end. A 60 kg person walks from S2 toward the right end. How far past S2 can the person walk before the plank tips? (Use g = 9.8 m/s^2.)



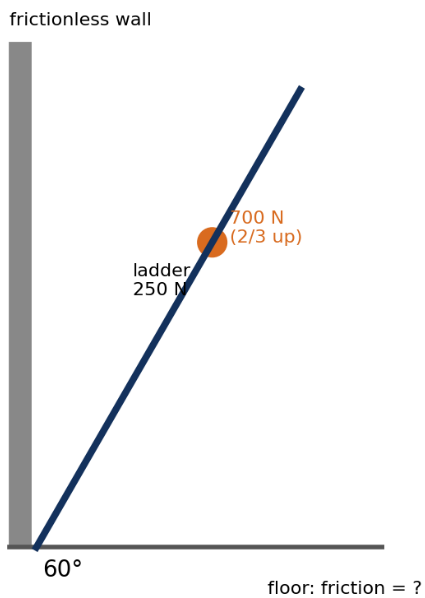
Problem 4 [Rotational equilibrium]

A uniform horizontal strut of weight 150 N and length 3.0 m is hinged to a wall at its left end. A cable from the right end runs up to the wall at 40° above the strut. A 500 N load hangs from the right end. Find the cable tension and the magnitude of the hinge reaction force.



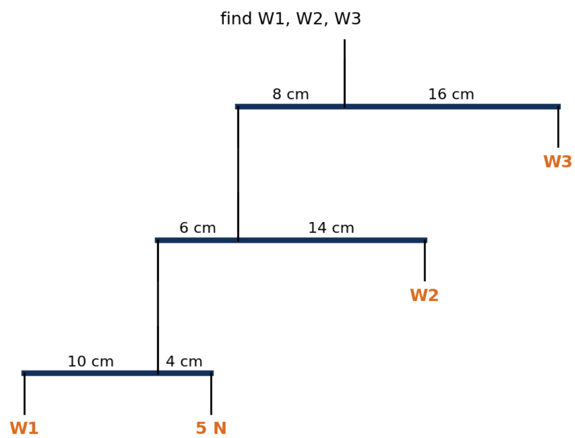
Problem 5 [Rotational equilibrium]

A uniform ladder of weight 250 N leans at 60° above the horizontal against a frictionless wall. A 700 N person stands two-thirds of the way up. The floor provides friction. Find the minimum coefficient of static friction between ladder and floor that keeps it from slipping.



Problem 6 [Rotational equilibrium]

The hanging mobile shown is in balance (ignore the mass of the rods and strings). Working from the bottom bar upward, find the unknown weights W_1, W_2, and W_3.

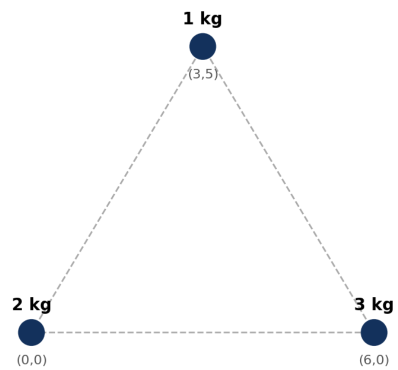


Problem 7 [Center of mass]

Three point masses lie on the x-axis: 4 kg at x = 0, an unknown mass m at x = 3.0 m, and 2 kg at x = 8.0 m. The centre of mass of the three is at x = 2.8 m. (a) Find m. (b) A 5 kg mass is then added so the centre of mass of all four lands at x = 3.5 m. Where must it be placed?

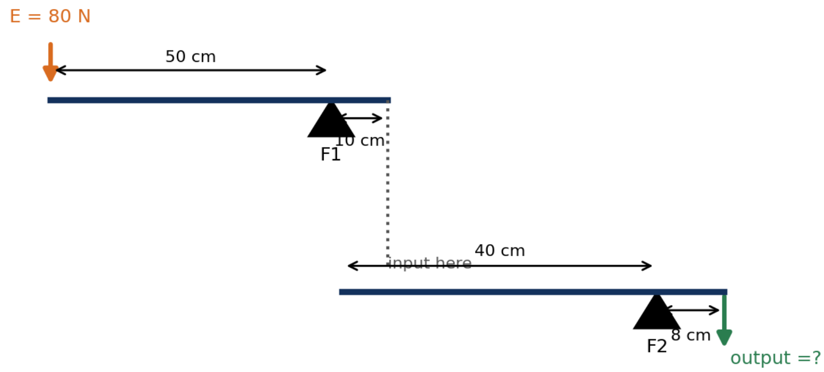
Problem 8 [Center of mass]

Point masses sit at the vertices of a triangle: 2 kg at (0, 0), 3 kg at (6, 0), and 1 kg at (3, 5) (metres). (a) Find the centre of mass. (b) A 4 kg mass is added so the new centre of mass of all four is at (3.0, 0). Find the coordinates where the 4 kg mass must be placed.



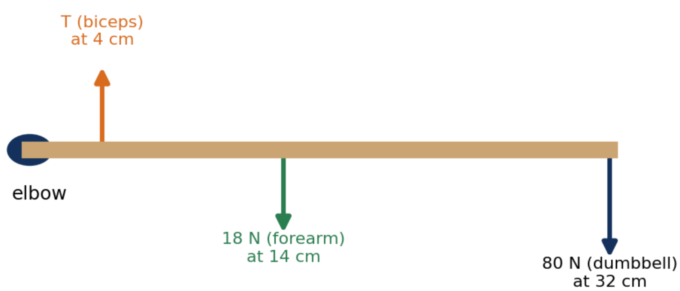
Problem 9 [Levers / mechanical advantage]

Two levers act in series. An 80 N effort is applied to lever 1, which has an effort arm of 50 cm and a load arm of 10 cm. The output of lever 1 drives lever 2, which has an effort arm of 40 cm and a load arm of 8 cm. Find the final output force and the overall mechanical advantage.



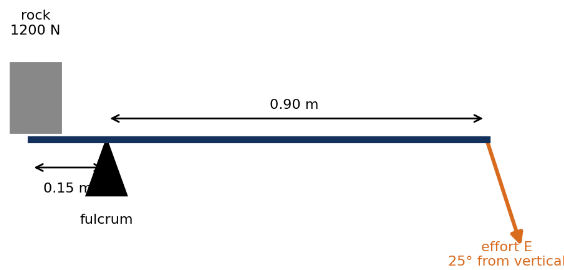
Problem 10 [Levers]

A forearm is held horizontal (see free-body diagram). The biceps attaches 4.0 cm from the elbow and pulls straight up. The forearm weighs 18 N with its centre of gravity 14 cm from the elbow, and an 80 N dumbbell is held 32 cm from the elbow. Find the biceps tension.



Problem 11 [Levers / torque]

A crowbar pries up a rock. The fulcrum is 0.15 m from the rock, which presses down on the short end with 1200 N. The effort is applied 0.90 m from the fulcrum, but its line of action is tilted 25° away from the perpendicular to the bar. Find the effort force required.


rock
1200 N
0.90 m
0.15 m
fulcrum
effort E
25° from vertical

Answer Key — Standard 3.1 Hard Set

Problem 1

Take counterclockwise as positive; lever arm = r·sin(angle to radius).

tau_1 = +30(0.40) = +12.0 N·m (tangent, CCW)

tau_2 = –50(0.40)(sin 30°) = –50(0.40)(0.5) = –10.0 N·m (CW)

tau_3 = –20(0.20) = –4.0 N·m (CW)

net = 12.0 – 10.0 – 4.0 = –2.0 N·m

Answer: Net torque = 2.0 N·m, clockwise.

Common error: Using the full 0.40 m arm for F_2 instead of 0.40·sin 30°. Only the component perpendicular to the radius makes torque.

Problem 2

Both unknowns are support forces. Use sum of torques = 0 about A (erases A), then sum of forces.

Torque about A: B(6.0) – 400(2.5) – 200(4.5) = 0

B(6.0) = 1000 + 900 = 1900 -> B = 316.7 N

Forces: A + B = 400 + 200 = 600 -> A = 600 – 316.7 = 283.3 N

Answer: A = 283 N, B = 317 N (both upward).

Common error: Putting the beam's weight at the centre (3.0 m). It acts at the centre of gravity, 2.5 m from A.

Problem 3

It tips about S2 when the force on S1 drops to zero. Take torques about S2 at that instant.

Plank weight = 30(9.8) = 294 N at 2.5 m (that is 1.5 m left of S2): holds it down, 294(1.5) = 441 N·m

Person weight = 60(9.8) = 588 N at distance d right of S2: 588·d

Tipping: 588·d = 441 -> d = 0.75 m

Answer: The person can walk 0.75 m past S2 (4.75 m from the left end) before it tips.

Common error: Measuring the person's distance from the left end instead of from the pivot S2. Torque uses distance from the pivot.

Problem 4

Take torques about the hinge (erases the hinge force). Only the cable's vertical part lifts.

Torque about hinge: T(sin 40°)(3.0) = 150(1.5) + 500(3.0) = 225 + 1500 = 1725

T = 1725 / (3.0·sin 40°) = 1725 / 1.928 = 895 N

Hinge: H_x = T cos 40° = 685 N; H_y = 650 – T sin 40° = 650 – 575 = 75 N (up)

Hinge magnitude = sqrt(685^2 + 75^2) = 689 N

Answer: Cable tension T = 895 N; hinge reaction = 689 N (685 N horizontal, 75 N up).

Common error: Forgetting the strut's own 150 N weight at its midpoint, or using the cable's full tension instead of its vertical component for the lift torque.

Problem 5

Wall is frictionless, so its force is purely horizontal. Take torques about the base.

Vertical: N_{floor} = 250 + 700 = 950 N

Torque about base (L cancels): N_{wall} sin 60° = 250(0.5)cos 60° + 700(2/3)cos 60°

N_{wall} = cot 60°·(125 + 466.7) = 0.5774(591.7) = 341.7 N

Friction needed f = N_{wall} = 341.7 N; mu = f / N_{floor} = 341.7 / 950

Answer: Minimum coefficient of friction mu = 0.36.

Common error: Giving the frictionless wall a vertical (friction) force. A frictionless surface can only push perpendicular to itself — here, horizontally.

Problem 6

Each bar balances on its own. Solve the bottom bar first, carry its total weight up.

Bottom bar: $W_1(10) = 5(4)$ -> $W_1 = 2$ N. Bottom total = 2 + 5 = 7 N

Middle bar: $7(6) = W_2(14)$ -> $W_2 = 3$ N. Middle total = 7 + 3 = 10 N

Top bar: $10(8) = W_3(16)$ -> $W_3 = 5$ N

Answer: $W_1 = 2$ N, $W_2 = 3$ N, $W_3 = 5$ N.

Common error: Forgetting to carry the FULL weight of a finished bar (rod load + both hanging weights) up to the next level.

Problem 7

Centre of mass is the weighted average of position.

(a) $(4\cdot0 + m\cdot3 + 2\cdot8)/(4 + m + 2) = 2.8$ -> $(3m + 16) = 2.8(6 + m)$

$3m + 16 = 16.8 + 2.8m$ -> $0.2m = 0.8$ -> $m = 4$ kg

(b) Existing moment = $4\cdot0 + 4\cdot3 + 2\cdot8 = 28$, total mass 10

$(28 + 5p)/(10 + 5) = 3.5$ -> $28 + 5p = 52.5$ -> $p = 4.9$ m

Answer: (a) m = 4 kg. (b) Place the 5 kg mass at x = 4.9 m.

Common error: Dividing by the number of masses instead of the total mass, or forgetting to add the new mass to the denominator in (b).

Problem 8

Compute weighted averages separately for x and y.

(a) $x = (2\cdot0 + 3\cdot6 + 1\cdot3)/6 = 21/6 = 3.5$; $y = (2\cdot0 + 3\cdot0 + 1\cdot5)/6 = 5/6 = 0.83$

(b) Current moments: $M_x = 21$, $M_y = 5$; total mass 6, add 4 kg -> total 10

x: $(21 + 4x)/10 = 3.0$ -> $4x = 9$ -> $x = 2.25$

y: $(5 + 4y)/10 = 0$ -> $4y = –5$ -> $y = –1.25$

Answer: (a) cm at (3.5, 0.83). (b) Place the 4 kg mass at (2.25, –1.25).

Common error: Treating the y-target of 0 as 'ignore y'. A target of zero still gives an equation that fixes the y-position.

Problem 9

For levers in series, multiply the mechanical advantages (ratios of arms).

$MA_1 = 50/10 = 5$ -> output of lever 1 = 80(5) = 400 N

$MA_2 = 40/8 = 5$ -> final output = 400(5) = 2000 N

Overall MA = 2000/80 = 25 (= $MA_1 \times MA_2$)

Answer: Final output force = 2000 N; overall mechanical advantage = 25.

Common error: Adding the two mechanical advantages instead of multiplying them. Series machines multiply.

Problem 10

Take torques about the elbow; the joint force then drops out.

$T(0.04) = 18(0.14) + 80(0.32)$

$T(0.04) = 2.52 + 25.6 = 28.12$

$T = 28.12 / 0.04 = 703$ N

Answer: Biceps tension = 703 N (far larger than the load — a class-3 lever trades force for range).

Common error: Ignoring the forearm's own 18 N weight, or using cm instead of m and getting the arm ratios wrong.

Problem 11

Only the component of the effort perpendicular to the bar makes torque, so the angle costs you.

Useful effort = E cos 25° at 0.90 m; load torque = 1200(0.15) = 180 N·m

E cos 25°(0.90) = 180

E = 180 / (0.90·cos 25°) = 180 / (0.90·0.9063) = 180 / 0.8157 = 221 N

Answer: Effort required = 221 N (vs 200 N if pushed perpendicular — the 25° tilt wastes effort).

Common error: Multiplying by cos 25° in the wrong place or ignoring the angle entirely. Tilting the effort lengthens the force but shortens its lever arm.